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The Effects of a Graphing-Approach Intermediate Algebra Curriculum on Students' Understanding of Function

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In this study, we extended O'Callaghan's computer-intensive algebra study by using his component competencies and the process-object framework to investigate the effects of a graphing-approach curriculum employing the TI-82 graphing calculator. We found that students in the graphing-approach classes demonstrated significantly better understanding of functions on all 4 subcomponents of O'Callaghan's Function Test, including the reification component, than did students in the traditional-approach classes. Additionally, no significant differences were found between the graphing-approach and traditional classes either on a final examination of traditional algebra skills or on an assessment of mathematics attitude.

Key Words: Algebra; Conceptual knowledge; Curriculum; Functions; Graphing calculators

The function concept is considered by many to be one of the most central concepts in all mathematics, yet it is one for which students rarely develop adequate understanding. Numerous studies (Harvey, Waits, & Demana, 1995; Mayes, 1995; Ruthven, 1990) have been conducted to examine the effects of graphing technologies and graphing curricula on function understanding. These studies were generally focused on student achievement and generally showed that students using graphing technology performed as well on tests of traditional algebra as did students without such technology and, at the same time, improved performance on visual and graphing tasks. Kieran (1993) called for a different type of research in which we

> do more than simply show that technology can help our students to learn such material. We have to think about committing ourselves to very detailed studies of human cognition in this domain. The process/object theoretical framework can, at the very least, help to tie such findings together. (p. 223)

By *process/object framework* Kieran was referring to the ontological duality of functions; that is, a function can be thought of in two ways: operationally (as a process) and structurally (as an object).

This article is based on the first author's dissertation completed at North Carolina State University under the direction of the second author.

One researcher who did incorporate a process-object duality framework into his work was O'Callaghan (1998). O'Callaghan studied the effects of the *Computer-Intensive Algebra* (CIA) (Fey, 1992) curriculum on college algebra students' understanding of the function concept by comparing students using the CIA with students who experienced a traditional curriculum. He developed a function test, diagnostic in nature, to assess students' understanding of functions. Each question on the test was designed to assess one of the following aspects of conceptual knowledge of functions without the use of the graphing calculator: (a) modeling a real-world situation using a function, (b) interpreting a function in terms of a realistic situation, (c) translating among different representations of functions, and (d) reifying functions. *Reification* refers to the transition from the operational to the structural phase of concept development. O'Callaghan (1998) found that the CIA students achieved a better overall understanding of functions than traditional students and were better at modeling, interpreting, and translating, but he found no differences for reification, the most difficult of the four aspects of function knowledge. In addition, O'Callaghan administered the Revised Mathematics Attitude Scale (Aiken, 1972; Dutton, 1962) and found that students in the CIA curriculum significantly improved their attitudes toward mathematics over the semester.

Our main purpose in this study was to extend O'Callaghan's CIA study by using his framework to investigate a different curriculum, specifically a graphing-approach curriculum employing the TI-82 graphing calculator. We wanted to see if his results on the four components of his function test and on the Revised Mathematics Attitude Scale would hold within a different curriculum incorporating graphing calculators. Of particular interest was the reification component. Kieran (1992) reported that although students rarely acquire any real sense of the structural aspects of algebra, graphing software might help to develop structural conceptions. We wanted to determine whether the graphing curriculum along with the graphing calculator facilitated reification of the function concept. In addition, we wanted to examine student performance on a test of traditional algebra skills to determine the influence of the graphing curriculum.

METHODOLOGY

Participants

The participants in the study were college students enrolled in intermediate algebra. The sample was taken from students enrolled in the intermediate algebra course at a large state university with approximately 28,000 students. Students enrolled in intermediate algebra were those students scoring lowest on the university's mathematics placement examination. A total of 90 students participated in the study: 46 in the treatment group and 44 in the control group.

Treatments

The college text *Intermediate Algebra: A Graphing Approach* (Hubbard & Robinson, 1995) was used in conjunction with TI-82s in the treatment classes. A balance of graphing calculator and traditional algebra work is found in the text, which includes exploration and discovery examples to help guide students to look for patterns and make discoveries. Use of the TI-82 enabled students to explore, estimate, and discover graphically and to approach problems from a multirepresentational perspective. The students had access to the calculators both in class and for homework exercises and tests but not for the O'Callaghan Function Test or the traditional final examination.

The textbook used in the control classes, *Intermediate Algebra: Concepts and Applications,* fourth edition (Bittinger, Keedy, & Ellenbogen, 1994), covered the same topics as the experimental text but emphasized memorizing isolated facts and procedures and becoming proficient with paper and pencil calculations. It focused on simplifying and transforming expressions and solving equations. The control group had no known graphing calculator access.

Instrumentation

The function test developed and used by O'Callaghan (1995, 1998) was administered to all students in this study both as the pretest at the beginning of the semester and as a posttest at the end of the semester. The test is designed to be administered without access to graphing calculators. It is diagnostic in nature in that each question is designed to assess one of the following aspects of conceptual knowledge of functions: (a) modeling a real-world situation, (b) interpreting a function in terms of a realistic situation, (c) translating among different representations of functions, or (d) reifying functions.

The instrument chosen to evaluate students' traditional algebra skills was the departmental final examination, a 50-question test of conventional algebra skills. It was administered to all four classes during the final week of the semester, and students were allowed 3 hours to complete it.

To measure students' attitudes toward mathematics, we administered the Revised Mathematics Attitude Scale (Aiken, 1972; Dutton, 1962) both before and after treatment, as did O'Callaghan. This instrument has been used in numerous studies and is considered to be one of the measures of choice regarding attitudes toward mathematics.

Research Design

The performance of students who had used the TI-82 in a graphing-approach curriculum was compared with the performance of students who had been in a traditional algebra curriculum, using the instruments described above. Four sections of a semester-long intermediate algebra course were used in a balanced design with two instructors each teaching one experimental and one control class. One of each of two simultaneous morning sections and two simultaneous afternoon sections were randomly selected to use the experimental curriculum. Students registered for classes by telephone via a computerized scheduling program. Class populations were expected to be similar. Students in the experimental sections were given the opportunity to switch to the control section being taught at the same time, but none elected to do so.

The researchers observed each of the experimental and control classes on a random basis throughout the semester, focusing mainly on teachers' behavior and lesson development and on students' behavior and calculator use. For each treatment, the instructors planned together and followed the same plans of study, adhering to the course syllabus. From interviews and observations the researchers concluded that the instructors were not biased in favor of either approach.

Descriptive statistics and analysis of variance (ANOVA) procedures were performed on both the O'Callaghan function pretest results and the demographic variables to determine any initial differences among the four classes in the study. Class profiles of typical characteristics provided data about students' genders (as measured by the percentage that were males), average ages, mathematical backgrounds (number of previous algebra courses), ability in mathematics (Math SAT scores), predicted grade-point average in mathematics (PGM) calculated by departmental formula, and verbal ability (Verbal SAT scores). Analysis of these characteristics indicated that the students in the four classes were different only with respect to gender composition. There were more female students overall and more males than females in the experimental group. Analysis of the function pretest scores indicated no significant differences among the four classes on prior knowledge of functions, which therefore was not used as a covariant in the final analysis.

RESULTS

O'Callaghan Function Test

Students' understanding of the function concept was analyzed initially using a MANOVA on the four component scores and the total score on the function test, and this analysis was supplemented by univariate results on the individual components. Table 1 shows the means and standard deviations by treatment for each component and for the total score.

Table 1
Function Posttest Mean Scores

		Experimental		Control	
Component	Maximum score	*M*	*SD*	*M*	*SD*
Modeling	7	4.32	1.65	3.33	1.64
Interpreting	11	7.46	1.92	5.90	2.21
Translating	9	5.05	2.26	3.64	2.21
Reifying	10	4.20	1.89	2.74	0.29
Total	37	21.02	5.87	15.62	4.70

The experimental classes had higher means for each of the four components as well as for the total score on the function test. The MANOVA, $F(4, 69) = 4.68$, revealed an overall significant treatment effect at the $\alpha = .01$ level, indicating that the experimental classes had a significantly better overall understanding of functions than the control classes had. Univariate results of ANOVAs on the four components revealed significant differences at the $\alpha = .05$ level in favor of the graphing-approach group for modeling, interpreting, and translating, and at the $\alpha = .01$ level for reifying.

Departmental Final Examination

Although the experimental group overall had a slightly higher mean score on this examination (used to measure the students' traditional algebra skills), no significant difference was found between the scores of treatment and control classes. Similarly, although one instructor's students had a slightly higher mean score than the other instructor's students, no significant differences were found for main effects of instructor or gender or for any interactions among the three variables.

Attitude Survey

The attitude survey posttest, conducted after students had completed the algebra course except for the departmental final exam, showed that students in the experimental class had slightly more positive attitudes than their counterparts in the control class had about mathematics and their mathematical abilities. There was, however, no significant difference between the experimental and control classes, between instructors, or between the genders.

CONCLUSIONS AND DISCUSSION

Function Test

Because of the availability of graphing calculators, the graphing-approach curriculum can include examples and problems for modeling real-world situations with functions that would be either too time-consuming or impractical without a graphing calculator. The graphing calculator affords the user both the ability to create equations, tables, and graphs quickly and the facility to move among the representations rapidly. Thus, it can be concluded that the graphing-approach students who used the TI-82 were more comfortable than the traditional students when working with real-world data and situations. The experimental group had become accustomed over the semester to examining functions from different perspectives and accordingly performed significantly better than the traditional students on interpreting and translating questions. The results on the first three components are in agreement with O'Callaghan's findings with the CIA curriculum.

On the reification component, however, we found a significant difference between the graphing-approach group and the control group, whereas O'Callaghan found no difference. The reification component scores, which were the lowest of any for the four components for both the traditional and experimental groups in both studies, indicate the difficulty of the reification process, a process that involves a much greater degree of abstraction than the other three aspects of function knowledge. Reification is not a process that can be taught. Instead, it is the shift involved in making the transition from an operational to a structural understanding of a concept. In O'Callaghan's study of the CIA curriculum, students had access to graphing technology only in a lab setting. The graphing-approach students in the present study had access to the graphing-calculator during every class meeting as well as for homework and so had more opportunities to explore functions and to examine abstract applications. This difference in access to graphing technology may account for the differences in reification found for the two groups in this study but not found in the O'Callaghan study.

No significant differences in traditional algebraic skills were found between the experimental and control groups. It can therefore be inferred that the experimental students who used graphing calculators for the semester were not hindered in their computational ability. The graphing-calculator treatment was not expected to give the students any advantage over the traditional students because the final examination focused mainly on paper-and-pencil calculations and manipulations such as simplifying and transforming symbolic expressions and solving equations. These results are somewhat congruent with the findings in O'Callaghan's (1998) study. Although O'Callaghan found a lower level of proficiency in symbolic manipulation with the CIA students than with his traditional students, he found no significant differences after adjusting for the CIA students' initial lower mathematical competence.

We found that students in the graphing-approach curriculum as a group did not differ from traditional students in their attitudes toward mathematics, but O'Callaghan (1998) found that students using the CIA curriculum significantly improved their mathematical attitudes over the course of the semester. As yet, there is no consensus on the effect of technology use on attitude.

Recommendations for Further Research

It has been argued that "of all the areas where further research could profitably be carried out, the one that seems to stand out as being clearly in need of attention is that of finding ways to develop structural conceptions in students" (Kieran, 1992, p. 413). A continued research focus is needed to help find ways to facilitate the transition from operational to structural conceptions in students. Research on the reification of functions and other concepts should be expanded. Studies are needed to advance the knowledge of how structural and procedural conceptions interact when students are doing algebra within a technological environment. It is important to study how technology positively and negatively affects the development of both structural and procedural conceptions.

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